

Massive or massless scalar field and confinement

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Received: 15 November 2002 / Revised version: 31 January 2003 /

Published online: 7 March 2003 – © Springer-Verlag / Società Italiana di Fisica 2003

Abstract. The generalization of QCD motivated classical $SU(2)$ Yang–Mills theory coupled to a scalar field is discussed. The massive scalar field, corresponding to the scalar glueball, provides a confining potential for static, point-like, external sources. In the case of a massless scalar field screening solutions are found. However, there is a confining sector as well. Both, massive and massless confining solutions, are compared with phenomenological potentials. The case of a non-dynamical permittivity is also discussed.

1 The model

Recently, Dick and Fulcher [1] have proposed an interesting model in which the lowest glueball, represented by a massive scalar field ϕ , has been coupled in a non-minimal way to the $SU(2)$ gauge fields. They have chosen, in analogy with the chiral quark models [2], one glueball coupling: $\phi F^{a\mu\nu} F_{\mu\nu}^a$. Their model provides confinement of quarks and gives a quite reasonable interquark potential, $U_{q\bar{q}} \sim r^{1/3}$.

However, as was pointed out by Zalewski and Motyka, the most probable potential which gives the best fit to the quarkonium system takes a different form [3], namely

$$U_{MZ}(r) = C_1 \left(\sqrt{r} - \frac{C_2}{r} \right), \quad (1)$$

where $C_1 \simeq 0.71 \text{ GeV}^{1/2}$ and $C_2 \simeq 0.46 \text{ GeV}^{3/2}$. Due to this the model should be modified. This can be done by using a slightly more complicated effective coupling. It has to be underlined that in spite of that modification the scalar field still represents the lowest scalar glueball.

In the present paper we focus on the following scalar–gauge action:

$$S = \int d^4x \left[-\frac{1}{4} \frac{\left(\frac{\phi}{\Lambda}\right)^{8\delta}}{1 + \left(\frac{\phi}{\Lambda}\right)^{8\delta}} F_{\mu\nu}^a F^{a\mu\nu} + \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{m^2}{2} \phi^2 \right]. \quad (2)$$

Here, m is the mass of the scalar field, Λ is a dimensional and δ a dimensionless constant. In order to have the standard Maxwell-like behavior of the fields in the neighborhood of sources the particular form of the denominator has been added. It does not deflect the long range behavior of the fields but provides asymptotic freedom in the limit of a strong scalar field [4].

The field equations corresponding to the action (2) take the form

$$D_\mu \left(\frac{\left(\frac{\phi}{\Lambda}\right)^{8\delta}}{1 + \left(\frac{\phi}{\Lambda}\right)^{8\delta}} F^{a\mu\nu} \right) = j^{a\nu}, \quad (3)$$

and

$$\partial_\mu \partial^\mu \phi = -2\delta F_{\mu\nu}^a F^{a\mu\nu} \frac{\phi^{8\delta-1}}{\Lambda^{8\delta} \left(1 + \frac{\phi^{8\delta}}{\Lambda^{8\delta}}\right)^2} - m^2 \phi, \quad (4)$$

where $j^{a\mu}$ is an external current.

In the next section it will be shown that the action (2) possesses confining solutions in the massive sector. Moreover, these solutions describe a wide family of confining potentials which can be compared with various potentials found in fits to the phenomenological data [5–7].

Interestingly enough, there is also a confining solution in the massless sector, where the scalar field cannot be identified with any glueball. Finally we prove that the model with a non-dynamical scalar field provides confinement of the external sources as well. The problem of glueball states in these models is discussed in the last section.

2 Confining solutions

2.1 Massive case

Let us now discuss the Coulomb problem. We find solutions of the field equations generated by an external, static, point-like source:

$$j^{a\mu} = 4\pi q \delta(r) \delta^{a3} \delta^{\mu 0}. \quad (5)$$

The restriction to the Abelian source is not essential. It is possible to investigate a more general, static non-Abelian

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source [8]: $j_\mu^a = 4\pi q\delta(r)C^a\delta^{\mu 0}$. Here C^a is the expectation value of the $su(N_c)$ generator for a normalized spinor in color space. However, the fields generated by the above sources have an identical dependence on the spatial coordinates. They differ only by a multiplicative color factor. For that reason it is sufficient to analyze only an Abelian source (5). Then the field equations read

$$\left[r^2 \frac{\left(\frac{\phi}{\Lambda}\right)^{8\delta}}{1 + \left(\frac{\phi}{\Lambda}\right)^{8\delta}} E \right]' = 4\pi q\delta(r), \quad (6)$$

$$\nabla_r^2 \phi = -4\delta E^2 \frac{\phi^{8\delta-1}}{\Lambda^{8\delta} \left(1 + \frac{\phi^{8\delta}}{\Lambda^{8\delta}}\right)^2} + m^2 \phi. \quad (7)$$

We use the standard definition $E^{ai} = -F^{a0i}$. The electric field is chosen in the same color direction as the source $\vec{E}^a = E(r)\delta^{3a}\hat{r}$. One can easily derive the electric field from (6) and express it in terms of the scalar field:

$$E(r) = \frac{q}{r^2} \left(1 + \left(\frac{\phi}{\Lambda}\right)^{-8\delta}\right). \quad (8)$$

This field can be understood as the usual Coulomb field $E(r) = \alpha_{\text{eff}}/r^2$ in an unusual medium. The medium is characterized by the scalar field which modifies the effective coupling constant α_{eff} :

$$\alpha_{\text{eff}} = q \left(1 + \left(\frac{\phi}{\Lambda}\right)^{-8\delta}\right). \quad (9)$$

Using (8) we are able to rewrite the equation (7) in the following form:

$$\nabla_r^2 \phi = -4\delta \frac{q^2}{\Lambda r^4} \left(\frac{\phi}{\Lambda}\right)^{-8\delta-1} + m^2 \phi. \quad (10)$$

Unfortunately, this equation is still too complicated to integrate it analytically. However, on account of the fact that we are mainly interested in the long range behavior of the fields we can analyze (10) in the asymptotic regime, i.e. for $r \rightarrow \infty$. The asymptotic solution is found to be

$$\phi \sim \left(\frac{2\sqrt{\delta}q\Lambda^{4\delta}}{m}\right)^{1/(1+4\delta)} \left(\frac{1}{r}\right)^{2/(1+4\delta)}. \quad (11)$$

Then the electric field is

$$E \sim \frac{q}{r^2} + \left(\frac{4\delta q}{m^2}\right)^{-4\delta/(1+4\delta)} \Lambda^{8\delta/(1+4\delta)} \left(\frac{1}{r}\right)^{(2-8\delta)/(1+4\delta)}. \quad (12)$$

Finally, the corresponding potential has the form

$$U \sim -\frac{q}{r} + \frac{1+4\delta}{12\delta-1} \left(\frac{4\delta q}{m^2}\right)^{-4\delta/(1+4\delta)} \times \Lambda^{8\delta/(1+4\delta)} \left(\frac{1}{r}\right)^{(1-12\delta)/(1+4\delta)}, \quad (13)$$

for $\delta \neq -1/4$ or $1/12$. One can observe that $\delta = 1/12$ corresponds to a logarithmic behavior of the electric potential:

$$U \sim -\frac{q}{r} + \left(\frac{q}{3m^2}\right)^{-1/4} \Lambda^{1/2} \ln r. \quad (14)$$

In the framework of such classical models, confinement is understood as the appearance of a singularity of the electric potential (energy density) at spatial infinity. The energy becomes infinite due to the long range behavior of fields. This effect takes place for $\delta \geq 1/12$. However, it was shown by Seiler [9] that confining potentials cannot grow faster than linearly for large r . This gives us an upper bound for the parameter δ . Concluding, the model given by the action (2) can be used for modelling confinement of the external sources for the following parameter δ :

$$\delta \in \left[\frac{1}{12}, \frac{1}{4}\right]. \quad (15)$$

The standard linear potential is obtained for $\delta = 1/4$. Then we achieve a model discussed previously by Dick [4]. In this model a global, confining solution was found. As one could expect the asymptotic behavior of the solution is linear.

2.2 Massless case

In this section we are going to focus on the problem of confinement in the case of a massless scalar field. Apparently, such a field can no longer be identified with the scalar glueball. Nonetheless, one can still use it to modify the properties of the vacuum. The scalar field becomes an effective field describing dynamical permittivity of the medium, i.e. the vacuum. One can notice that the model belongs to the so-called color dielectric field theories.

Equation (10) for the scalar field takes a simpler form now:

$$\nabla_r^2 \phi = -4\delta \frac{q^2}{\Lambda r^4} \left(\frac{\phi}{\Lambda}\right)^{-8\delta-1}, \quad (16)$$

which can be integrated analytically. We have found a family of solutions regular at spatial infinity and labelled by a positive parameter β_0 :

$$\phi = A\Lambda \left(\frac{1}{r\Lambda} + \frac{1}{\beta_0}\right)^{1/(1+4\delta)}, \quad (17)$$

where the constant A is given by

$$A = [q(1+4\delta)]^{1/(1+4\delta)}.$$

The corresponding electric field has the form

$$E = \frac{q}{r^2} + A^{-8\delta} \frac{q}{r^2} \left(\frac{1}{r\Lambda} + \frac{1}{\beta_0}\right)^{-8\delta/(1+4\delta)}. \quad (18)$$

We see that there exists an infinite set of electric field solutions, generated by the external point source, which falls as r^{-2} for large r . The behavior of the electric field

near a source strongly depends on the value of the parameter δ . If $\delta \geq 0$ the singularity at $r = 0$ is identical to that generated by a point source in the standard Maxwell electrodynamics. If $\delta < 0$ then the singularity can be arbitrarily large.

Besides the family of solutions mentioned above, there is a single field configuration, which solves the equations of motion and gives confinement. The solution is

$$\phi = A\Lambda \left(\frac{1}{r\Lambda}\right)^{1/(1+4\delta)}. \quad (19)$$

The electric field is then given by the formula

$$E = \frac{q}{r^2} + A^{-8\delta} q \Lambda^2 \left(\frac{1}{r\Lambda}\right)^{2/(1+4\delta)}. \quad (20)$$

Thus the electric potential for $\delta \neq 1/4$ takes the form

$$U = -\frac{q}{r} + \frac{4\delta + 1}{4\delta - 1} A^{-8\delta} q \Lambda^{8\delta/(1+4\delta)} \left(\frac{1}{r}\right)^{(1-4\delta)/(1+4\delta)} \quad (21)$$

and for $\delta = 1/4$

$$U = -\frac{q}{r} + q A^{-8\delta} \Lambda \ln(\Lambda r). \quad (22)$$

In fact, we observe confining behavior (in the same sense as for the massive field) for the parameter δ :

$$\delta \in \left[\frac{1}{4}, \infty\right). \quad (23)$$

As was recently shown [10], it is possible to get rid of the non-confining solutions (17) and (18) by adding the following potential term:

$$V(\phi) = \alpha \phi^4 \left(\frac{\phi}{\Lambda}\right)^{8\delta}, \quad (24)$$

where α is a positive constant. Then only the confining solution survives. Quite interesting, if we omit the denominator in the dielectric function in the action (2) (this means that we are no longer interested in restoration of the standard r^{-2} behavior of the electric field in a small neighborhood of a source) then the non-confining solutions describe the so-called screening phenomena known from quantum Yang–Mills theory [11]. The field configuration generated by a fixed charge can have an arbitrary small but positive energy [12].

2.3 Non-dynamical permittivity

Let us now proceed to discuss the third possibility, that is, to neglect the kinetic term of the scalar field in (2). In this case ϕ is no longer a dynamical field [13]. In other words, it is possible to treat it as an additional field which, after expressing it in terms of the gauge fields, can be completely removed from the action. One can easily observe that in

order to deal with a non-trivial theory we are forced to add a potential term for the scalar field. In our calculation the potential is chosen as in the massless case (24).

Then the action takes the following form:

$$S = \int d^4x \left[-\frac{1}{4} \left(\frac{\phi}{\Lambda}\right)^{8\delta} F_{\mu\nu}^a F^{a\mu\nu} - \alpha \phi^4 \left(\frac{\phi}{\Lambda}\right)^{8\delta} \right]. \quad (25)$$

After variation with respect to the scalar field one gets

$$8\delta F \left(\frac{\phi}{\Lambda}\right)^{8\delta-1} - 4(2\delta + 1)\alpha \phi^4 \left(\frac{\phi}{\Lambda}\right)^{8\delta-1} = 0, \quad (26)$$

where $F = -(1/4)F^{a\mu\nu}F_{\mu\nu}^a$. As previously we will investigate only the electric part of the theory. Then one can trivially solve it and find

$$\phi^4 = \frac{E^2}{a}, \quad (27)$$

where for simplicity a new constant $a = (2\delta + 1)\alpha$ was defined. Substituting this into the Gauss law we obtain

$$\left[r^2 \left(\frac{E^2}{a\Lambda^4}\right)^{2\delta} E \right]' = 4\pi q \delta(r). \quad (28)$$

The solution reads

$$E = a^{2\delta/(1+4\delta)} \left(\frac{|q|}{\Lambda^2 r^2}\right)^{1/(1+4\delta)} \Lambda^2. \quad (29)$$

This gives us the electric potential

$$U = a^{2\delta/(1+4\delta)} |q|^{1/(1+4\delta)} \frac{4\delta + 1}{4\delta - 1} \left(\frac{1}{\Lambda r}\right)^{(1-4\delta)/(1+4\delta)} \Lambda \quad (30)$$

for $\delta \neq 1/4$ and

$$U = \sqrt[4]{aq^2} \Lambda \ln(\Lambda r) \quad (31)$$

for $\delta = 1/4$. Identically to the massless scalar field the confinement of the point sources is found for all δ parameter values equal to or larger than $1/4$. From the confinement point of view these models are equivalent.

As was said before, the action (25) can be expressed only by gauge fields. After a simple calculation one obtains the Pagels–Tomboulis lagrangian [14]:

$$L = -\frac{1}{4} F^2 \left[\left(\frac{F^2}{\Lambda^4}\right)^2 \right]^\delta. \quad (32)$$

Of course, the pertinent equations of motion, in the electric sector, derived for the Pagels–Tomboulis model are the same as (28) (up to an unimportant multiplicative constant).

Models in which the permittivity depends on the strength tensor F has been very successfully analyzed in the context of the effective theory for low energy QCD [15, 16]. Here no additional, effective scalar field is needed. Confining solutions emerge due to the modification of the vacuum by the gauge fields. In fact, in the Pagels–Tomboulis model a source charge generates an infinite energy field configuration, whereas a configuration originating from a dipole-like source has finite energy [14, 17].

3 Conclusions

In the present paper we have analyzed the $SU(2)$ gauge field coupled to the scalar field. For the massive and massless scalar field as well as for the non-dynamical scalar field, solutions describing confinement have been found. It is relatively easy to model confinement of external sources using the scalar dilaton/glueball field. Our models seem to be very similar from the confinement point of view. Of course, there are also some differences between them.

Firstly, the occurrence of confinement strongly depends on the value of the parameter δ . The massive and massless models require different values of δ . For example the best phenomenological quark–antiquark potential, found by Zalewski and Motyka in fits to the spectrum of quarkonia, is obtained for $\delta = 3/20$ for the massive and for $\delta = 3/4$ for the massless case respectively. In fact, then $U \sim \sqrt{R}$, where R is the distance between the sources. Another known interquark potential, $U \sim R^{0.1}$, used by Martin [7], is realized for $\delta = 11/116$ (massive field) or for $\delta = 11/36$ (massless field). The difference between these models is even more apparent in the case of linear confinement. The massive model describes it for $\delta = 1/4$. It is unlikely for the massless model, where the linear potential is realized only in the limit $\delta \rightarrow \infty$, which cannot be implemented on the action level. The linear confinement can be only approximated with arbitrary accuracy by taking a sufficiently large δ .

To conclude, in spite of the differences mentioned above, all the models considered here can very well serve to model confinement of quarks. Nevertheless these models differ essentially. One of the most profound differences is visible if we look at the way how glueball states appear in the models.

If we would like to treat these models as candidates for the description of low energy QCD, the problem of glueballs must be taken into consideration. It is obvious that a good effective model is expected not only to provide confinement of quarks (and give a reasonable quark–antiquark potential) but is also expected to have solutions which could be interpreted as glueballs. In the massive case the situation is clear. The scalar field represents the scalar glueball 0^{++} which has mass m . The dielectric function determines the coupling between the glueball and gluons. For instance, $\delta = 1/8$ gives one glueball coupling. For different values of δ we have a more complicated effective coupling. Unfortunately, this model is unable to describe other glueball states. In order to do this one has to enlarge the number of fields. For example the pseudo-scalar glueball 0^{-+} , represented by a new field ψ , requires an axion-like coupling [18].

The situation changes drastically if we consider the remaining models (with the dynamical or non-dynamical scalar field). Obviously, the interpretation that the scalar

field corresponds to the glueball is no longer correct. Because of that, glueballs should appear in a different, non-trivial way. It is believed that in the framework of these models glueballs could be described as (topological) solitons i.e. static, localized and finite energy solutions of the sourcesless field equations. This is a very attractive way of studying glueballs. Different glueballs would be given by different solitons characterized by a topological index (cf. e.g. the Faddeev–Niemi model [19]).

As a result, the existence of topologically non-trivial solutions in the framework of scalar–gauge models is crucial from the QCD point of view. It would let us justify which model can be really relevant as the effective model for low energy quantum chromodynamics. We plan to address this problem in our proceeding paper.

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